Voting Rules: The Median Voter Model

The Median Vote Model: A Numeric Example with Direct Democracy

Eleven voters are voting on spending levels for a Local School. They already know what form of taxation will be used to pay for it, and all other relevant information.

Assume that:
- All voters are Fully Informed and Rational
- All voters will in fact Vote, for the budget that is closest to their preferred position
- Dis-utility from spending more or less than the preferred amount is equal
- The spending level that gets the most votes will win
- Everybody votes their immediate interests (no strategic voting)

Note, later models will all explore the above assumptions in more detail.

We can map out the preferences of our eleven voters as below.

Eleven Voters, arranged on a High spending – Low spending axis (Left – Right)

<table>
<thead>
<tr>
<th>Voter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired Spending</td>
<td>20</td>
<td>17</td>
<td>13</td>
<td>12</td>
<td>11</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Assume now that we vote on spending levels, that two proposals are put forward, either randomly (by lot) or from an Agenda Setter.

What does Frank vote for?

**Weak Median voter theorem:** Any policy that the median voter favors will defeat any other policy that it is competing with:

- Example – if current spending is 4, and somebody proposes to spend 8 instead, 8 will defeat 4.
- Voters A,B,C,D,E,F,G,H will all vote for 8, only I,J,K will vote for 4

Whenever a vote comes up, if Frank supports it, it passes, if Frank opposes it, it fails. Thus, over time, every vote either fails (Frank opposes it), or wins because it moves our spending level closer to what Frank supports, and thus votes for.

**Strong Median voter theorem:** The policy preferred by the median voter will eventually win.

- Example – we have reached a spending level of 8. A new proposal will be made, and either fail, or it will be to spend 9. A,B,C,D,E,F will support it, G,H,I,J,K will oppose it. Spending is increased from 8 to 9, by 6 votes to 5. Frank gets what Frank wants

**Some Implications**
- The median voter gets what he wants, eventually.
- Extremism is not an issue.
  - Alan decides he wants a 40 million baht school
  - Ken decides on a 1 million baht school
    - Nothing has changed, whatever Frank wants, Frank gets.
    - “moderate” and median are the same.
- When the median voters preferences change, so does the policy.
- Frank decides he wants to spend 10 million on schools.
  - No policy can beat that in a pairwise election
- The only time other voters matter, is if one of them changes from one side of F to the other
Gary decides that he wants to spend 10 million on schools
Again, no policy can beat 10 million in a pairwise election
The median voter gets what he wants, but now Gary is the median voter
Movement or preference changes by other voters does not matter, unless they cross over the median voter

The median voter model and representative Democracy.
Each voter, rather then directly voting for a position, votes for a candidate who promises to spend at a certain level. Each position, at least initially, generates a candidate.

Proportional Representation. Candidates get votes in parliament proportionate to the number of votes they receive from the public. (generally by having more representatives). So if we have 11 seats in parliament to fill, nothing differs from our above results.

What if only candidates who can get at least 10% of the vote are allowed into parliament?

In that case, you need at least two voters to vote for your party. If only one voter votes for a party, the party does not reach the minimum threshold, and that voter is not represented at all.
The above model predicts a rightwing party consisting of I,J,K, and another party consisting of G,H, and maybe F. It predicts a leftwing party of A,B and maybe C. Another party will exist on the left, consisting of C,D or D,E, or possibly E,F (it is hard to predict)
Parties will always consist of at least two voters. Parties will rarely be larger than 3 voters. If a candidate (party) represents 4 voters, those voters have an incentive to vote for two different candidates, increasing their clout in parliament. Candidates have an incentive to fill that need, and the party splits or factionalizes. The party structure will be unstable.

Implications of Proportional voting systems in the Median Voter Model.
They will create multiple parties, as long as they can exceed the threshold of votes necessary.
Parties will be unstable. Any large party will be subject to splits. Large parties will have factions.

Extreme views will tend to be represented in Parliament. They may be over-represented.

Once in Parliament, parties will form coalitions. The median party gets what it wants.
The median party can/will form coalitions opportunistically
And will be seen as opportunistic
Politics can be positive, negative or zero sum

Extended analysis of Thai Political parties…………………..

First Past the Post (plurality) Voting In first past the post voting, whoever gets the most votes, wins.

This will lead to a different party structure; one with larger parties. Imagine the parties we had from the previous example: A,B,C and D,E,F and G,H and I,J,K. In this situation, a coalition of either ABC and DEF (majority), or DEF and GH (controlling minority), will dominate.

Now, the only candidate who wins is the one who gets the most votes. If ABC and DEF each voter for their own candidate (say B and E), GH and IJK can merge, and win the election. Candidate I could get 5 votes, while B and E each only received 3. Voter I wins, even though he only received 5 of eleven votes. Furthermore, this party does not contain the median voter. But now, parties ABC and DEF have an incentive to combine, because if they do so, they will have 6 votes, and win. But DEFGH has an incentive to attract F, so that they have a majority.

Implications of Winner Take all Voting systems in the Median Voter Model.
In theory a plurality of votes will win
In practice, only a majority is not threatened by a larger plurality
You will have two large, stable parties – center left and center right
The parties will talk about how extreme the other one is
But in fact both will tend to be moderate
Extreme positions are not well represented, or sometimes represented at all
It will be hard to distinguish between candidates, since they are both appealing to moderates
Third parties are spoilers – they hurt their own cause
Politics is Zero sum
Only with geographic parties will you get more than two parties for any length of time.

Extended analysis of U.S. Political Parties…………………

**BAS, SPD and LAW students are not responsible for anything after this**

Graphical Representations of the Median voter model.

Typically, we use frequency distributions, where one axis measures the number of voters, and the other measures policy on a left-right axis.

The left-right axis goes back to the French Revolution
Radicals (reformers) sat on the left side of the xxxxx
Monarchists (conservatives) sat on the right side of the xxxxxx

Since then, the “left” has come to mean Communists, socialists, Christian Democrats, Labor
The “right” means conservatives, monarchists, European liberals, Free marketers

Or in the U.S.,
Democrats (socialists, or U.S. Liberals) and Republicans (Conservatives)

Or in Thailand,
the Liberals and Thai Rhak Thai (yellows and reds)


Graphing the individual voter

How can we interpret what voter’s want? We can graph our median voter as getting utility from government (the public good), he will receive the highest utility at his ideal point. The farther from that point, the less utility he will have. Though we usually draw this utility curve (it has an alternate interpretation – willingness/likelihood of voting, more on that later) it does not have to be.

Example: our voter views govt. as a “safety net”, he wants it fund unemployment insurance. If he is risk adverse, he will view an underfunded govt. as less desirable than an overfunded govt.

having an indifference curve, where the peak is at his ideal point. Any deviation from that point will result in less utility.
Graphing the Full Electorate

This all assumes that voting can be modeled in two dimensions, and all voting is “single peaked”.

At the right is an example of a country with a fairly normal distribution of voters, most voters are somewhere in the middle, there is not a great deal of polarization, and policy/politics will not change much unless a large number of voters change their position.

See in class example of a change in the views of the Median voter.

In this example, voters are polarized between two worldviews. Depending on how/why the polarization exists, differing things could result

- Geographic polarization
  - Czechoslovakia 1989
- Ideological polarization
  - Spain 1936

See other in class examples

  - Policy will tend to be less stable, and a small number of voters switching views could lead to large changes in political orientation

Class discussion: Is Thai Politics the first or second case? Is U.S. politics the first or second case?

  - Negative advertising in Thailand and the U.S.
  - First past the post voting in Thailand and the U.S.

U.S.

Other Graphs (In Class) Bush vs. Gore; Clinton vs. Obama; Federal Germany, France Mitterand vs. LePen
Critiques of the Median voter model.

The median voter model is similar to many other economic theories, in that it predicts patterns of behavior, it does not predict winners and losers. This is fine for academics, but politicians (and businessmen) want to win elections (make money), this model doesn’t necessarily get them there.

More importantly, the median voter model makes some strong assumptions, that are in fact not completely realistic.

The problem of vote Cycling (Condorcet)

The median voter model assumes single peaked preferences along a single dimension
This works for many allocative economic decisions, such as roads or bridges
But non-economic decisions (or distributive ones) often can’t be defined this way

For example, imagine the U.S. after a terrorist attack. Policy makers (and by extension, the voters they represent) would fall into one of three categories

Hawks: It is their fault, so lets make them pay…. Conquer them, or if you can’t, Nuke them, or if you can’t, appease them.
Doves: It is our fault, we need to understand them and show them we love them Appease them, or conquer them, but don’t Nuke them…..
Isolationists: It is their fault, we must punish them, but who wants to run an Arab country? Nuke them, or appease them, or conquer them.

We have three voters, H,D and I
We have three policies, N, C, A

Our preferences for
H is C > N > A
D is A > C > N
I is N > A > C
Thus, a lower number is our preferred policy, higher numbers represent less preferred policies.

Q: In a pairwise vote, which policy will dominate?

Graphically, we can represent this in the graph to the Right. Note that our Isolationist has two policies that he prefers to conquest, either don’t get involved, or Nuke ‘em.

The Problem of Condorcet Vote Cycling II

Vote cycling also shows up when we are talking about distributive decisions, and note many decisions about providing a public good are in fact also distributive ones

<table>
<thead>
<tr>
<th>Voter</th>
<th>Utility</th>
<th>Hawks</th>
<th>Doves</th>
<th>Isolationists</th>
<th>Policy voted for</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>N will lose to C</td>
</tr>
<tr>
<td>Nuke them</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conquer them</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>C will lose to A</td>
<td></td>
</tr>
<tr>
<td>Appease them</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>A will lose to N</td>
<td></td>
</tr>
</tbody>
</table>

![Graph showing policy rank ordering with voter utility]
Case 1: Allocative efficiency

How can we use standard Indifference Curve Analysis to show an individual’s preferences for Public and Private Goods? In the analyses below, assume that our consumer is choosing between private consumption and a single, discrete, public good, in this case a bridge. Absent the ability to work with others, our consumer has an effective tax rate of 100% (t=1), and thus pays the full cost of the public good. With an IC curve that is tangent (IC_P), we get a corner solution, the consumer simply consumes other goods. See Graph A

- Many public goods, if not provided publically, end up as corner solutions, i.e. the market does not provide them, and consumers choose to not provide them for themselves.
- Other public goods can be provided, though they may or may not be.
- Examples: National Defense, local policing

In graph B, we have a different case. In this case, our consumer can cooperate with his neighbor. So he only has to pay half the cost of the bridge, but still gets the full benefits (a non-congestible public good), his new budget constraint is labeled t=.5. With the relevant IC(.5) indifference curve, he and his neighbor now consume more public good, but give up some private consumption to do so. As more neighbors get involved, consumption of both public and private goods increase when we go from a point where he pays 50% of the cost, to when he is only paying 33% of the cost.

Discussion:
- Under what circumstances will he not consume any public good (the corner solution)
- In going from t=1, to t=.5, and t=.33, is the public good a normal, superior, or inferior good?
- What about the private good?
- What are the relevant income and substitution effects?

Examples: Public Health, National Defense, Public Parks

Question: Are public goods a normal or superior good in the Aggregate? (see week 13, the growth of government)

In the above, the creation of a public good leads to allocative efficiency – by jointly providing the good, we increase our happiness, and that of the person who produces with us. This is Pareto Optimal

Case 2: Distributive consequences
Redraw Graph B, omitting the private good budget constraint and indifference curve (t=1 and ICP) for clarity. When our farmer needs to pay for half the cost of the bridge, his preferred point is b, for a total amount of our public good (the bridge) of G1. At this point, the MB\[G]/MB\[X] = MC\[G]/MC\[X], and he is consuming the optimal quantity of public and private goods. He would be indifferent between this point, (b), and either point a or point c, however at points a and c, he could be made better off by either substituting some public for private good, or vica versa.

We can redefine our original utility function U(Xa,G) and U(Xb,G) into terms t and G.

\[ X_{a} = Y_{a} - tG \]
\[ X_{b} = Y_{b} - (1-t)G \]

The above equations state that the amount of private goods we have is a function of our income less the amount of public goods we have, and how much we were taxed to obtain them. Any income that is not taxed to pay for public goods, purchases private goods.

Now
\[ U_{a} = U_{a}(Y_{a} - tG, G) \]
\[ U_{b} = U_{b}(Y_{b} - (1-t)G, G) \]

Explanation of graph – vertical axis is Private Good. So as t rises, the amount of private good that can be purchased decreases. The horizontal axis measures a public good.

Our budget constraint depends on the tax rate. As t goes down, our budget constraint shifts out (since A will be paying less taxes), if \( X_{a} \) and G are both normal goods, then consumption of both will increase, our indifference curve will shift up and rightward.

A couple of Notes:
If A cannot exchange with B, then his budget constraint is always t=1, i.e. he pays the full cost of the public good. This graph then is our standard indifference curve analysis.

As t goes down for A, he will demand more of the public good (price has gone down), he may or may not demand more of the private good (relative price of private to public goods has increased, but income has also increased). Much depends on the income and substitution effects of these changes.

(now, Graph of A’s tax-public good voting preference)
This model shows A’s preferred combination of public goods and private goods, for any given tax rate. Note, in this graph, a LOWER indifference curve is preferable, since it gives more of a good for less taxes….

Point 0, 1, and 2 correspond to the same point in the previous graph.

Let’s look in detail at these points. A is indifferent between 1 and 2, because the utility he gets from Xa,G at point 1 (only a little G, a lot of X) equals the utility he gets at 2 (a lot of G, but only a little X). However, at neither point is the MUxa = MUg, at point 1 the MUxa < MUg, at point 2 the MUxa > MUg

At point 0, we are at our optimal point, since MUxa = MUg

We can redefine our original utility function U(Xa,G) and U(Xb,G) into terms t and G.

\[ Xa = Ya - tG \]
\[ Xb = Yb - (1-t)G \]

The above equations state that the amount of private goods we have is a function of our income less the amount of public goods we have, and how much we were taxed to obtain them. Any income that is not taxed to pay for public goods, purchases private goods.

Now
\[ Ua = Ua(Ya - tG, G) \]
\[ Ub = Ub(Yb - (1-t)G, G) \]

We can map these in to a voter preference in a public good space, to obtain a contract curve

(Graph)…. 

Comments on the Graph:

This is a graph of points (G, and t (or (1-t))); and whether A and B would vote for those points.

A1 and B1 are the indifference curves of A and B if they have to pay the full cost of the public good themselves. No point above A1 would ever be acceptable to A, or a point below B1 acceptable to B
But points within this “eye” would be preferable, to both. Assuming G is a pure public good, they can both benefit by jointly producing it.