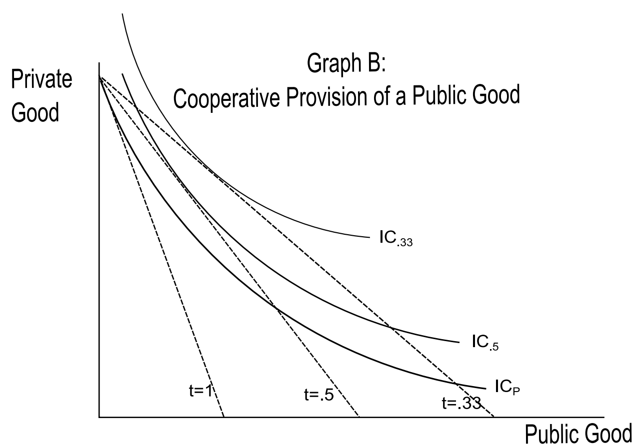
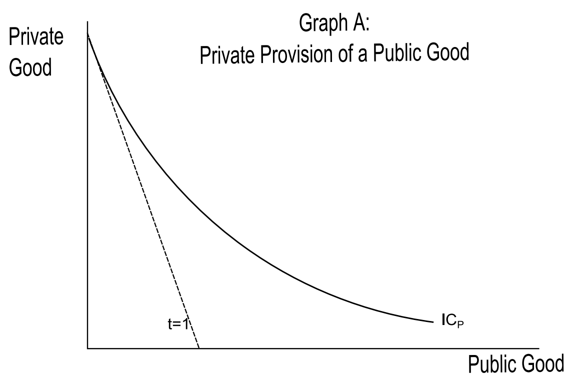
An Edgeworth Box with a Public Good
(the 'eye" graph)

Case 1: Allocative efficiency

How can we use standard Indifference Curve Analysis to show an individual's preferences for Public and Private Goods? In the analyses below, assume that our consumer is choosing between private consumption and a single, discrete, public good, in this case a bridge. Absent the ability to work with others, our consumer has an effective tax rate of 100% (t=1), and thus pays the full cost of the public good. With an IC curve that is tangent (IC_P), we get a corner solution, the consumer simply consumes other goods. See Graph A

- Many public goods, if not provided publically, end up as corner solutions, i.e. the market does not provide them, and consumers choose to not provide them for themselves.
- Other public goods can be provided, though they may or may not be.
- Examples: National Defense, local policing





In graph B, we have a different case. In this case, our consumer can cooperate with his neighbor. So he only has to pay half the cost of the bridge, but still gets the full benefits (a non-congestible public good), his new budget constraint is labeled t=.5. With the relevant IC.5 indifference curve, he and his neighbor now consume more public good, but give up some private consumption to do so. As more neighbors get involved, consumption of both public and private goods increase when we go from a point where he pays 50% of the cost, to when he is only paying 33% of the cost.

Discussion:

- Under what circumstances will he not consume any public good (the corner solution)
- In going from t=1, to t=.5, and t=.33, is the public good a normal, superior, or inferior good?
- What about the private good?
- What are the relevant income and substitution effects?

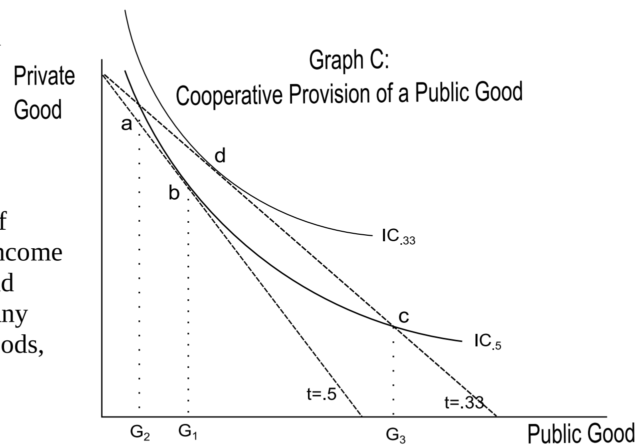
Examples: Public Health, National Defense, Public Parks

Question: Are public goods a normal or superior good in the Aggregate? (see week 13, the growth of government)

In the above, the creation of a public good leads to allocative efficiency – by jointly providing the good, we increase our happiness, and that of the person who produces with us. This is Pareto Optimal

Case 2: Distributive consequences

Redraw Graph B, omitting the private good budget constraint and indifference curve (t=1 and ICP) for clarity. When our farmer needs to pay for half the cost of the bridge, his preferred point is b, for a total amount of our public good (the bridge) of G1. At this point, the $MB_G/MB_X = MC_B/MC_X$, and he is consuming the optimal quantity of public and private goods. He would be indifferent between this point, (b), and either point a or point c, however at points a and c, he could be made better off by either substituting some public for private good, or vica versa.



We can redefine our original utility function U(Xa,G) and U(Xb,G) into terms t and G.

Xa = Ya – tG

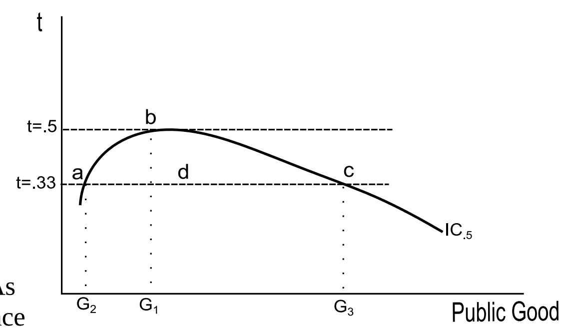
Xb = Yb – (1-t)G

The above equations state that the amount of private goods we have is a function of our income less the amount of public goods we have, and how much we were taxed to obtain them. Any income that is not taxed to pay for public goods, purchases private goods.

Now

Ua = Ua(Ya – tG, G)

Ub = Ub(Yb – (1-t)G, G)



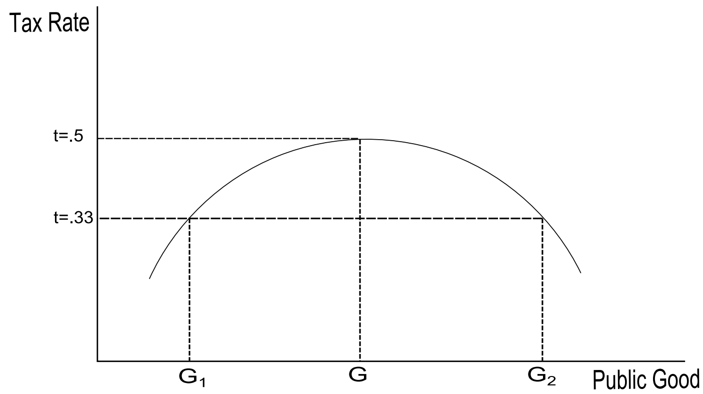
Explanation of graph – vertical axis is Private Good. So as t rises, the amount of private good that can be purchased decreases. The horizontal axis measures a public good.

Our budget constraint depends on the tax rate. As t goes down, our budget constraint shifts out (since A will be paying less taxes), if X_a and G are both normal goods, then consumption of both will increase, our indifference curve will shift up and rightward.

A couple of Notes:

If A cannot exchange with B, then his budget constraint is always t=1, i.e. he pays the full cost of the public good. This graph then is our standard indifference curve analysis.

As t goes down for A, he will demand more of the public good (price has gone down), he may or may not demand more of the private good (relative price of private to public goods has increased, but income has also increased). Much depends on the income and substitution effects of these changes.



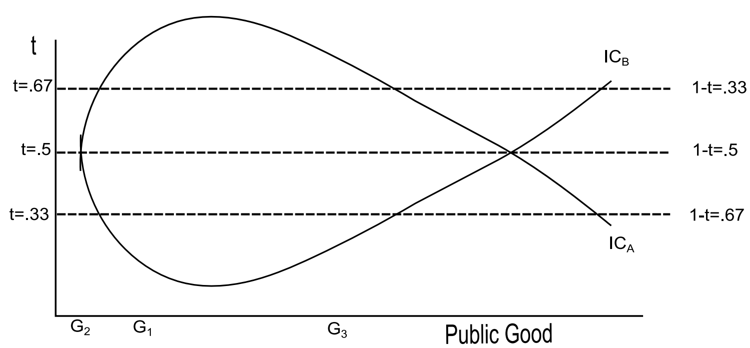
(now, Graph of A’s **tax-public good voting preference**)

This model shows A’s preferred combination of public goods and private goods, for any given tax rate. Note, in this graph, a LOWER indifference curve is preferable, since it gives more of a good for less taxes….

Point 0, 1, and 2 correspond to the same point in the previous graph.

Lets look in detail at these points. A is indifferent between 1 and 2, because the utility he gets from Xa,G at point 1 (only a little G, a lot of X) equals the utility he gets at 2 (a lot of G, but only a little X). However, at neither point is the MUxa = MUg, at point 1 the MUxa < MUg, at point 2 the MUxa > MUg

At point 0, we are at our optimal point, since MUxa = MUg



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Now
Ua = Ua(Ya – tG, G)

$Ub = Ub(Yb – (1\text{-}t)G, G)$

We can map these in to a voter preference in a public good space, to obtain a contract curve

(Graph out in class so students can work through it with me)….

Comments on the Graph:

This is a graph of points (G, and t (or (1-t)); and whether A and B would vote for those points.

A1 and B1 are the indifference curves of A and B if they have to pay the full cost of the public good themselves. No point above A1 would ever be acceptable to A, or a point below B1 acceptable to B

But points within this "eye" would be preferable, to both. Assuming G is a pure public good, they can both benefit by jointly producing it.

Consider point 1: (t1, G1) It is within the eye. A and B can both achieve higher utilities by adopting it.

The same for point 2 and 3, though these have radically different tax and level of public good implications.

Now consider point E. At this point, our indifference curves are tangent. It is impossible to move from this point to any other, because either A or B would be less well off if we did. But also note, at this point MU's between Public and private goods are not equal, B desires more public goods, A desires fewer of them. (for A, when the indifference curve has a negative slope, $MUg < MUxa$, the opposite for B.

Voting procedures:
Instead of just "randomly" offering points, and blindly groping forward; what if we direct the votes:

Instead of voting for a point, vote for a tax level; then consider all possible pairs of G that could be selected. Only if one is unanimously preferred to the others does t,G go into effect. If they can't go into effect, then select a new t, and start over. Now, we end up at Point L (for a Lindahl equilibrium)

Instead of voting blindly, imagine that a public servant proposes points. What kind of points will they propose?

General Voting rules: assume a central planner – then call out either taxes, or goods.

Critisisms of Unanimity rules:

Empirical Discussion: Two great objections to unanimity; transaction costs and strategic behavior.

Choosing the optimal Majority